G9 A.Math Notes

(1) Laws of Logarithm

(1) $\log_a b + \log_a c = \log_a(bc)$

(2) $\log_a b - \log_a c = \log_a\left(\frac{b}{c}\right)$

(3) $\log_a b^c = c\log_a b$

(4) $\log_a a = 1$

(5) $\log_a 1 = 0$

(6) $\log_a b = \dfrac{\log_c b}{\log_c a}$

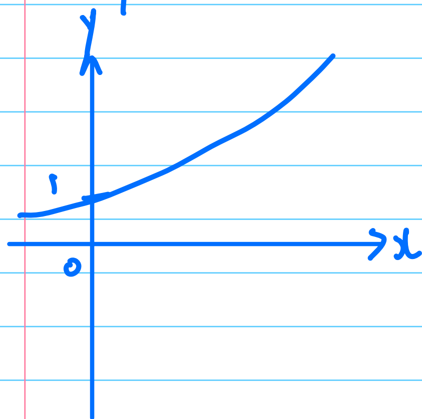
(7) $\log_a b = c$

$$\Downarrow$$

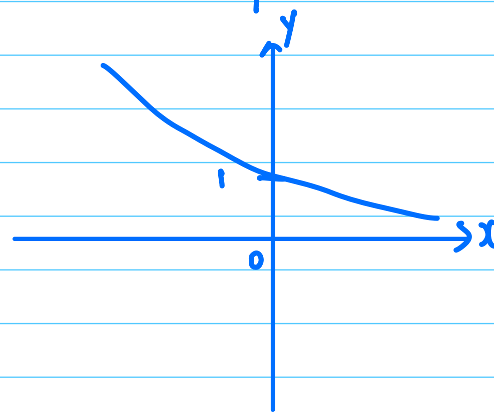
$$a^c = b$$

Note: $\ln x = \log_e x$ $\lg x = \log_{10} x$

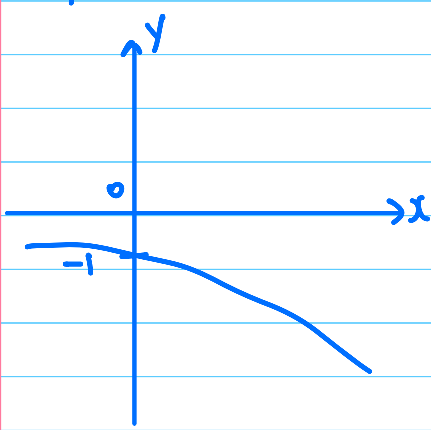
$y = e^{x}$



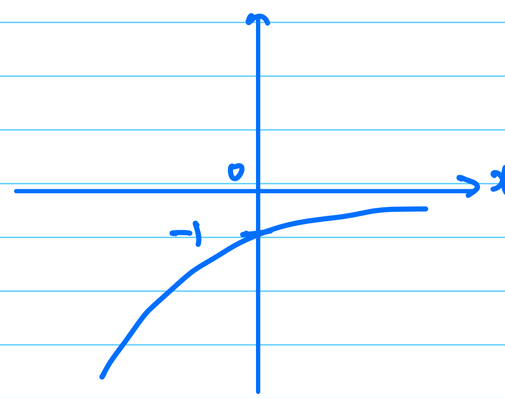
$y = e^{-x}$



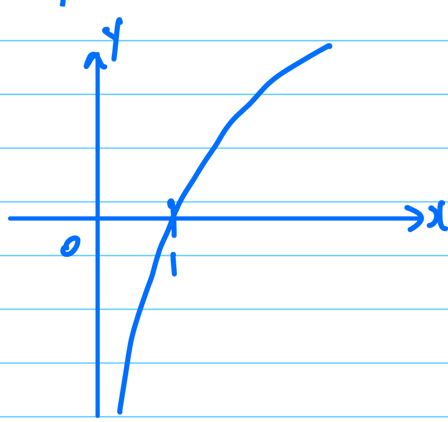
$y = -e^{x}$



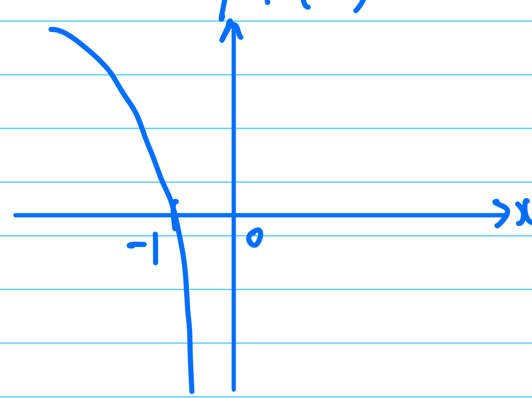
$y = -e^{-x}$



$y = \ln x$



$y = \ln(-x)$

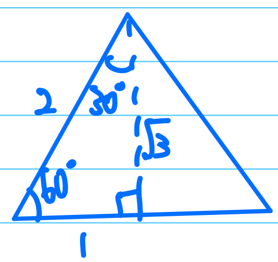


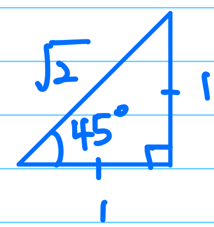
② Trigonometry (You have to know these for Math too)

$$\text{degree} \underset{\times \frac{180}{\pi}}{\overset{\times \frac{\pi}{180}}{\rightleftarrows}} \text{radians}$$

$180^\circ = \pi$ radians

$$\boxed{\begin{array}{llll} \text{Special angles:} & 30^\circ, & 45^\circ, & 60^\circ \\ & \frac{\pi}{6}\text{ rad} & \frac{\pi}{4}\text{ rad} & \frac{\pi}{3}\text{ rad} \end{array}}$$





$\sin 60^\circ = \frac{\sqrt{3}}{2}$

$\cos 60^\circ = \frac{1}{2}$

$\tan 60^\circ = \sqrt{3}$

$\sin 30^\circ = \frac{1}{2}$

$\cos 30^\circ = \frac{\sqrt{3}}{2}$

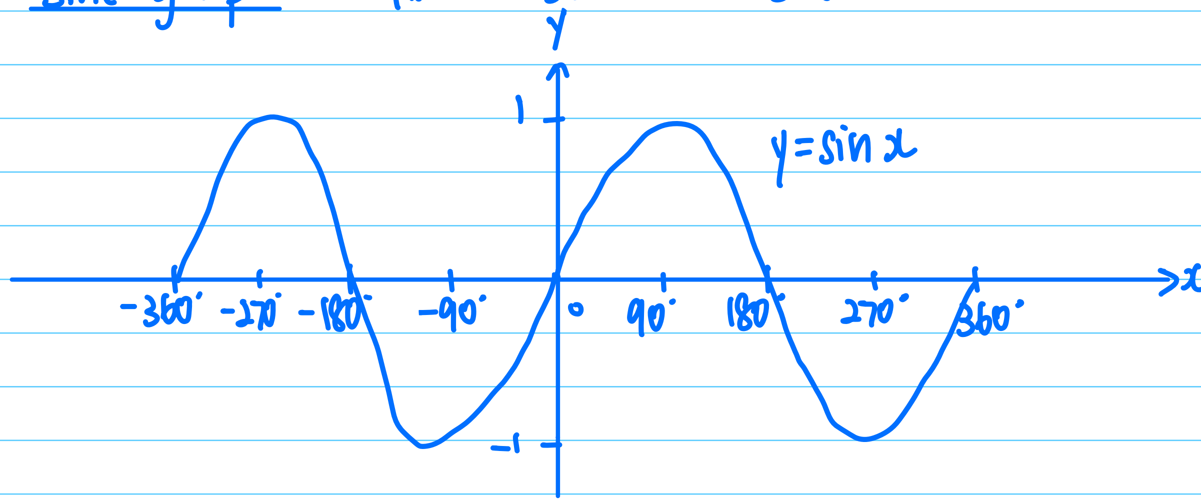
$\tan 30^\circ = \frac{1}{\sqrt{3}} \frac{\times\sqrt{3}}{\times\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 45^\circ = \frac{1}{\sqrt{2}} \frac{\times\sqrt{2}}{\times\sqrt{2}} = \frac{\sqrt{2}}{2}$

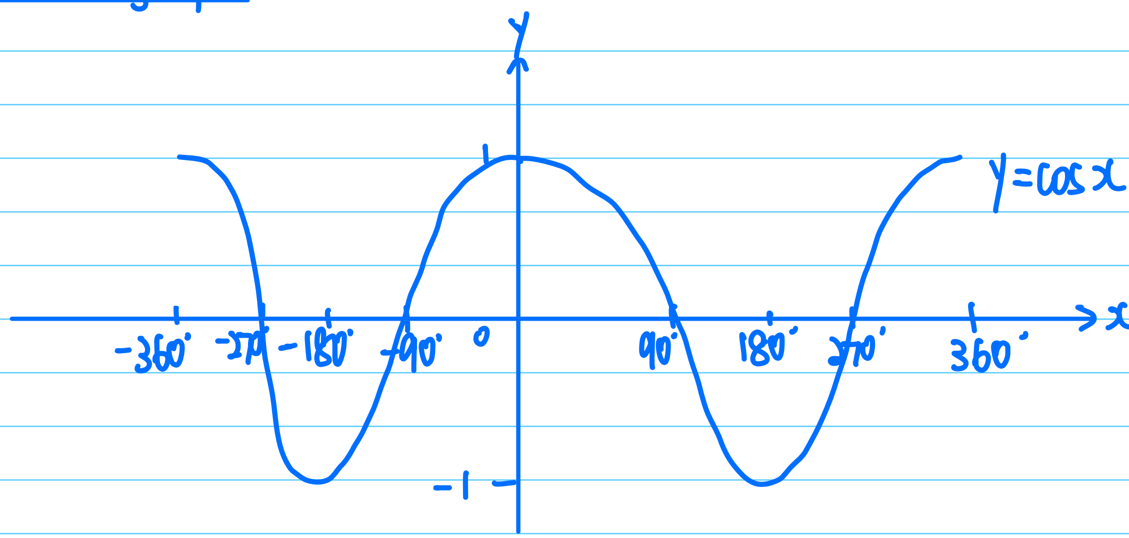
$\cos 45^\circ = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = 1$

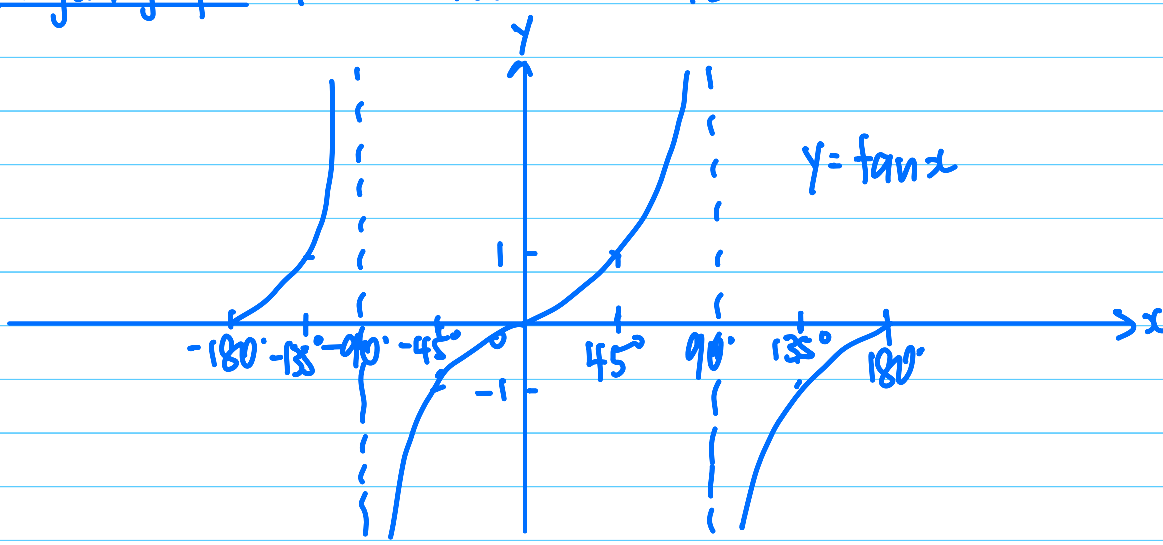
Sine graph for $-360^\circ \le x \le 360^\circ$



cosine graph for $-360^\circ \le x \le 360^\circ$



tangent graph for $-180^\circ \le x \le 180^\circ$



Other special angles : 0°, 90°

$0\,rad$, $\frac{\pi}{2}\,rad$

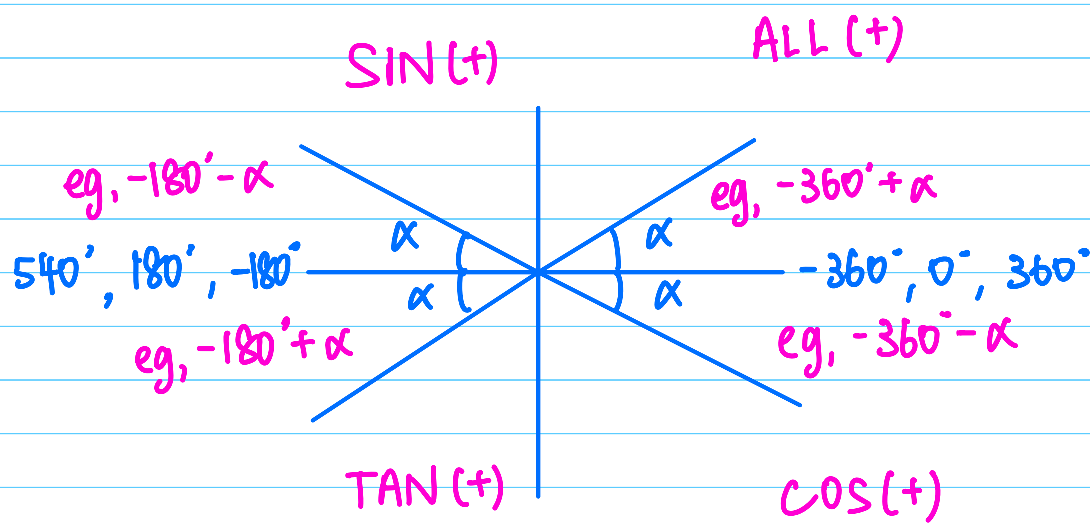
$\sin 0^\circ = 0$ $\qquad$ $\sin 90^\circ = 1$

$\cos 0^\circ = 1$ $\qquad$ $\cos 90^\circ = 0$

$\tan 0^\circ = 0$ $\qquad$ $\tan 90^\circ$ = no solution

You can get these from your graphs.

Quadrants :



eg. $\sin 120^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\tan 330^\circ = -\tan 30^\circ = -\frac{\sqrt{3}}{3}$

$\cos 135^\circ = -\cos 45^\circ = -\frac{\sqrt{2}}{2}$

→ $y = a\sin bx + c$ and $y = a\cos bx + c$

a = amplitude → $\dfrac{\text{maximum} - \text{minimum}}{2}$

$b = \dfrac{360^\circ}{\text{period}}$ or $\dfrac{2\pi}{\text{period}}$

period $= \dfrac{360^\circ}{b}$ or $\dfrac{2\pi}{b}$

- new max $= a(1) + c$
- new min $= a(-1) + c$

→ $y = a\tan bx + c$

$b = \dfrac{180^\circ}{\text{period}}$ or $\dfrac{\pi}{\text{period}}$

period $= \dfrac{180^\circ}{b}$ or $\dfrac{\pi}{b}$

Trigonometric Equations

1. $\sec x = \dfrac{1}{\cos x}$
2. $\operatorname{cosec} x = \dfrac{1}{\sin x}$
3. $\cot x = \dfrac{1}{\tan x} = \dfrac{\cos x}{\sin x}$
4. $\tan x = \dfrac{\sin x}{\cos x}$
5. $\sin^2 x + \cos^2 x = 1$
6. $\tan^2 x + 1 = \sec^2 x$
7. $\cot^2 x + 1 = \operatorname{cosec}^2 x$

given in the formula list. No need to memorise.

③ Permutations and Combinations

$$^nP_r = \frac{n!}{(n-r)!}$$

$$^nC_r = \frac{n!}{r!(n-r)!}$$

→ use P when order matters.

Eg, arranging books

standing in a straight line

password / code

→ use C when order does not matter.

Eg, choosing people for a team

choosing people for a committee

(4) Binomial Theorem

Expansion of $(a+b)^n$: $a^n + \binom{n}{1}(a)^{n-1}(b)^1 + \binom{n}{2}(a)^{n-2}(b)^2 + \ldots$

Finding a specific term:

$$\binom{n}{r}(a)^{n-r}(b)^r$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

All given in the formula sheet

5 $\boxed{\text{Series}}$

Arithmetic Progression: a = first term

d = common difference

nth term: $T_n = a + d(n-1)$

Sum of n terms: $S_n = \frac{n}{2}[2a + d(n-1)]$

Provided in formula sheet

Geometric Progression: a = first term

r = common ratio

nth term: $T_n = ar^{n-1}$

Sum of n terms: $S_n = \frac{a(1-r^n)}{1-r}$

Sum to infinity: $\frac{a}{1-r}$

↳ only exists if $|r| < 1$

Provided in formula sheet

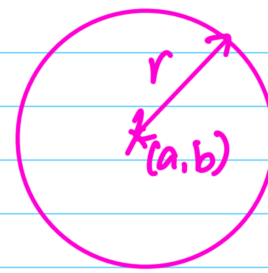
6 Equation of Circle

To find the equation of a circle, you need

1. coordinates of centre of circle
2. radius of circle

→ Equation of circle in completed square form

where centre = (a, b) radius = r

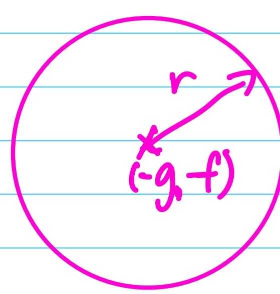


$$\boxed{(x-a)^2+(y-b)^2=r^2}$$

→ Equation of circle in general form:

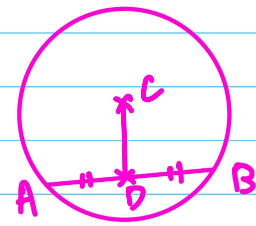
$$\boxed{x^2+y^2+2gx+2fy+c=0}$$

where centre = $(-g, -f)$ radius = $\sqrt{f^2+g^2-c}$



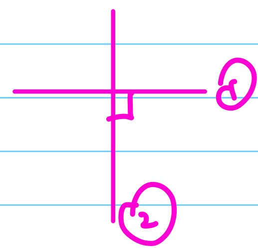
Note: 1 tan $\perp$ rad

2



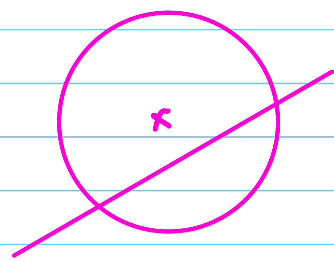
D is the mid-point of AB.

3



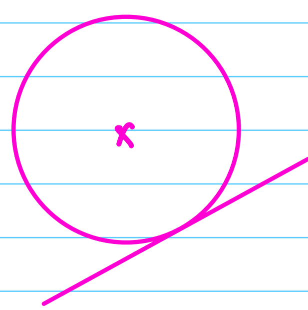
$m_1 \times m_2 = -1$

4



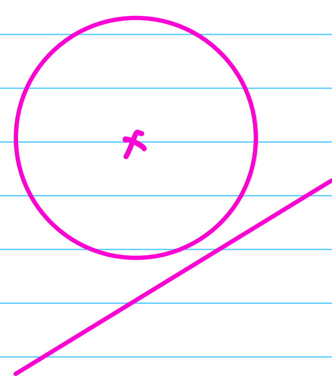
line intersects circle at 2 points :

$b^2 - 4ac > 0$



line is a tangent to the circle :

$b^2 - 4ac = 0$



line does not intersect circle :

$b^2 - 4ac < 0$

(7) Functions

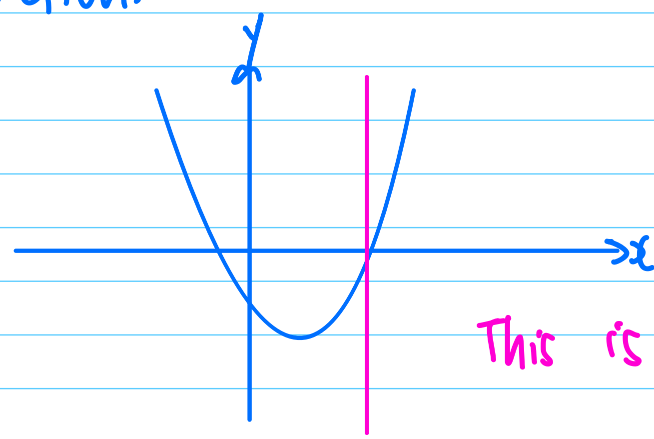
what defines a function?

An equation is a function if each x-value is mapped to only one y-value for a defined set of x values.

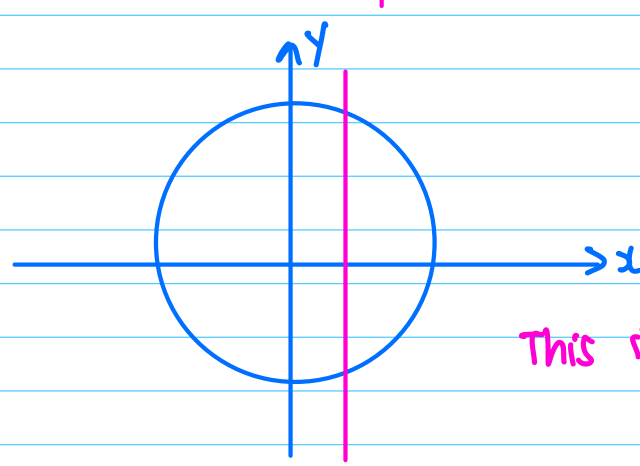
Each x value will only have ONE value of y.

Use the vertical line test to check if the relation is a function. If the vertical line intersects the graph in all places at exactly one point, the relation is a function.

eg,



This is a function.



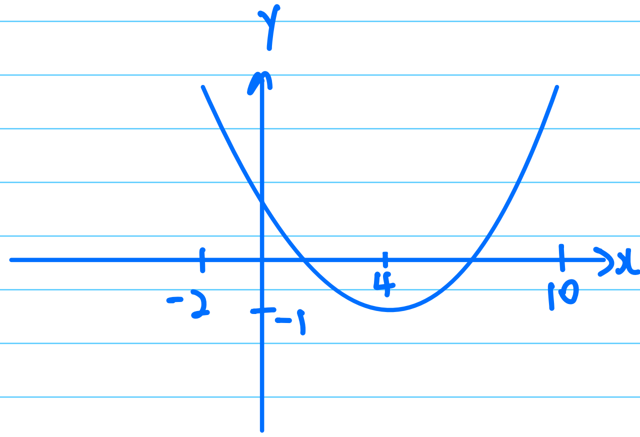
This is not a function.

Domain and range

Domain: range of x- values

Range: range of y- values

Eg,



Domain : $-2 \leq x \leq 10$

Range: $y \geq -1$

Composite functions : when one function is followed by another function

eg, $f(x) = 2x$
$g(x) = x^2 + 5$

$$gf(x) = (2x)^2 + 5$$
$$= 4x^2 + 5$$

$$fg(x) = 2(x^2 + 5)$$
$$= 2x^2 + 10$$

Inverse functions

$$\boxed{\begin{array}{l} \text{domain of } f(x) = \text{range of } f^{-1}(x) \\ \text{range of } f(x) = \text{domain of } f^{-1}(x) \end{array}}$$

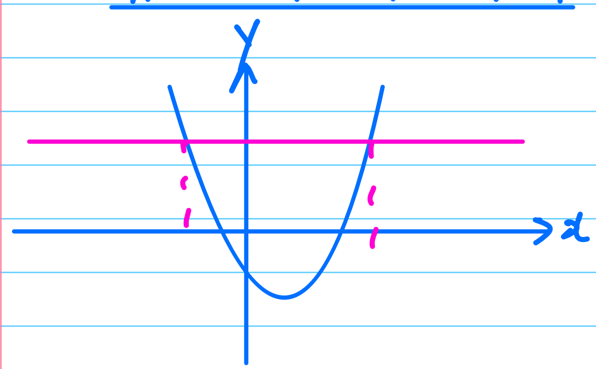
When does a function have an inverse?

↳ Only when each y-value is mapped to only 1 x-value for a defined set of y-values.

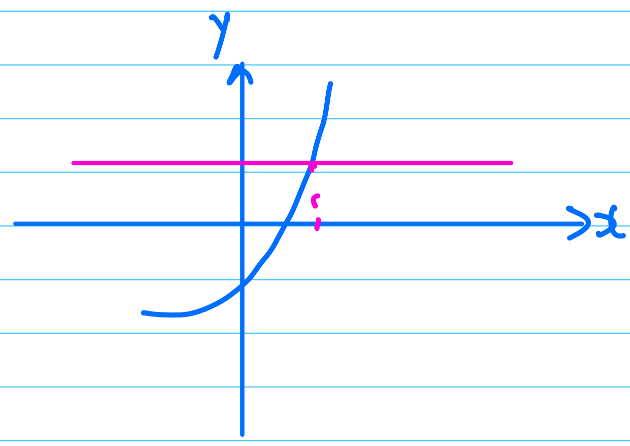
Each y-value will only have ONE value of x.

How to test if an inverse function exist?

horizontal line test → if the horizontal line intersects the graph at exactly one point in all places, an inverse of that function exists.



The inverse does not exist for this function.



The inverse exists for this function.

How to find the inverse of a function:

Eg, Given $f(x) = 5x-2$, find $f^{-1}(x)$.

let $y = f(x)$.

make x the subject: $y = 5x-2$

$$5x = y+2$$

$$x = \frac{y+2}{5}$$

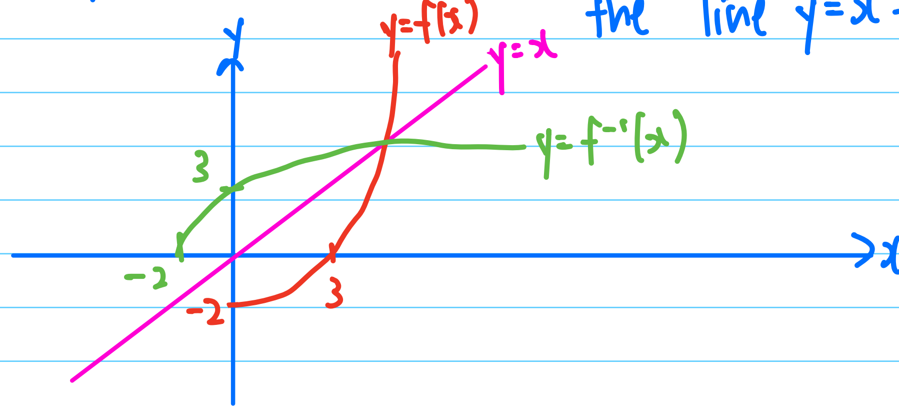
Replace x with y, and y with x:

$$y = \frac{x+2}{5}$$

$$\therefore f^{-1}(x) = \frac{x+2}{5}$$

Graphs of f and f^{-1}

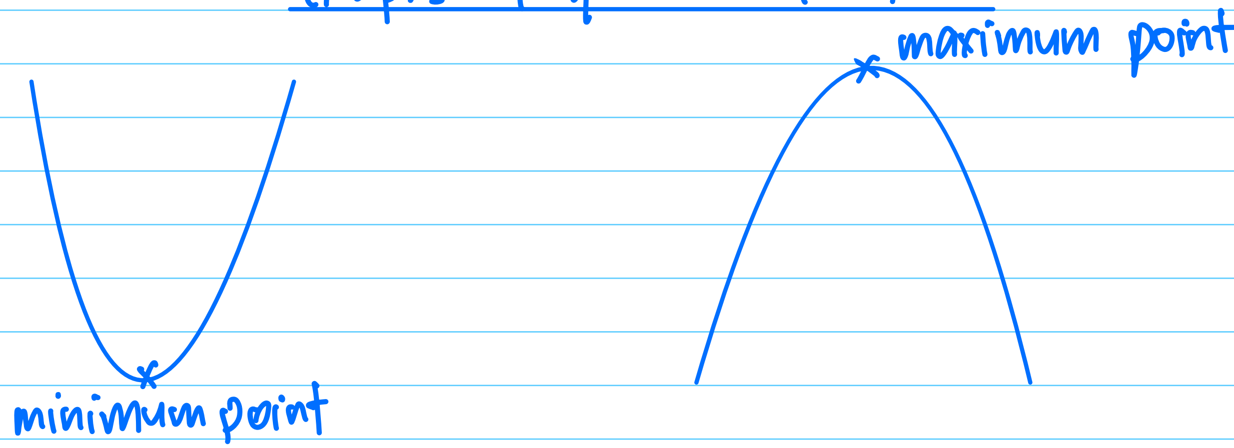
: Reflections of each other in the line $y = x$. (x becomes y; y becomes x)



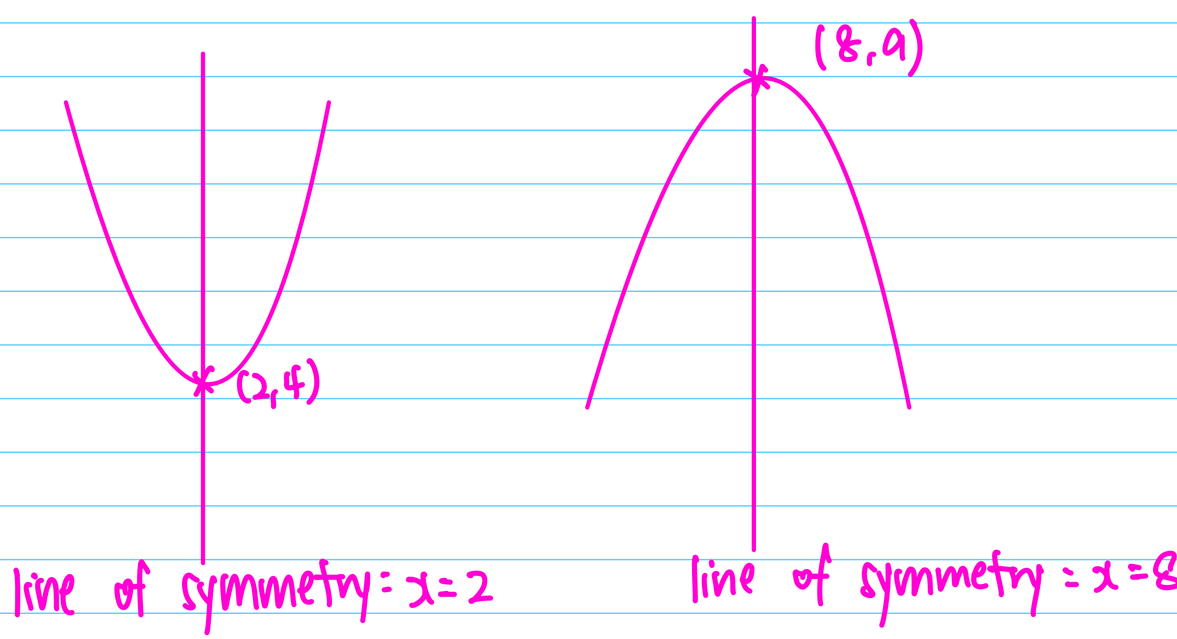
8 Simultaneous Equations and Quadratics

ax^2+bx+c is a quadratic function

Graphs of quadratic functions



The maximum and minimum points can also be called turning points or stationary points.



Complete the square

Example: Express x^2+8x in the form $(x+a)^2+b$.

$(x+4)^2-4^2$

$= (x+4)^2-16$

Express $x^2+10x-3$ in the form $(x+a)^2+b$.

$(x+5)^2-(5)^2-3$

$= (x+5)^2-25-3$

$= (x+5)^2-28$

$$\boxed{x^2+bx+c = \left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c}$$

Using $(x+a)^2+b$ to find turning point:

turning point $= (-a, b)$

Example: Given $(x-5)^2+6$,

turning point $= (5, 6)$

You need to know this for math as well.

Solving quadratic equations:

① Factorisation

Example: Solve $x^2 + 2x + 6 = 14$

$$x^2 + 2x + 6 - 14 = 0$$

$$x^2 + 2x - 8 = 0$$

$$(x+4)(x-2) = 0$$

$$x = -4 \quad \text{or} \quad x = 2$$

② Quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

provided in the formula list.

Example: Solve $3x^2 - 4x - 5 = 10$

$$3x^2 - 4x - 5 - 10 = 0$$

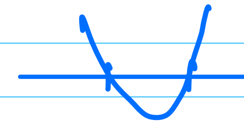
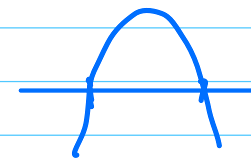
$$3x^2 - 4x - 15 = 0$$

$$a = 3,\ b = -4,\ c = -15$$

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(3)(-15)}}{2(3)}$$

$$= \frac{4 \pm \sqrt{16 + 180}}{6}$$

$$\therefore x = 3 \quad \text{or} \quad = -1\tfrac{2}{3}$$

When b^2-4ac	Nature of roots	Shape of curve
>0	2 real and distinct	 or 

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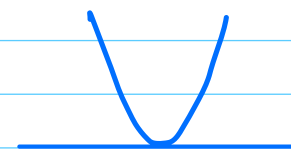
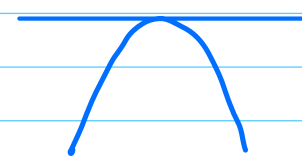
when b^2-4ac	Nature of roots	Shape of curve
$=0$	2 real and equal	 or 

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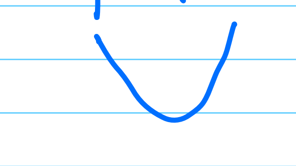
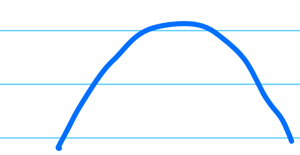
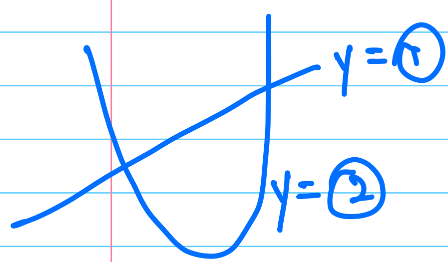
when b^2-4ac	Nature of roots	Shape of curve
<0	No real roots	 or 

When a line and a curve intersect :

1. line intersects the curve at **2 distinct points**

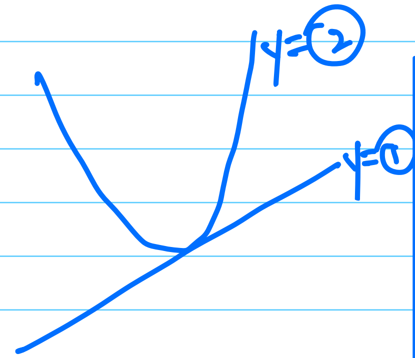


$1 = 2$

$1 - 2 = 0$

$b^2 - 4ac > 0$

2. line is a **tangent** to the curve

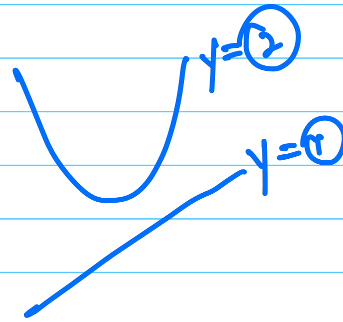


$1 = 2$

$1 - 2 = 0$

$b^2 - 4ac = 0$

3. line **does not intersect** the curve



$1 = 2$

$1 - 2 = 0$

$b^2 - 4ac < 0$

9 Factors and Polynomials.

Example : • If $x-2$ is a factor of $P(x)$,

$P(2)=0$

• If $3x-2$ is a factor of $P(x)$,

$P(\frac{2}{3})=0$

Example : • If the remainder is 5 when $P(x)$ is divided by $x-2$,

$P(2)=5$

• If the remainder is -3 when $P(x)$ is divided by $5x+1$,

$P(-\frac{1}{5})=-3$